Hale School
2010

Question/Answer Booklet

MATHEMATICS SPECIALIST 3CD
Section One
(Calculator Free)

Circle your teacher's initials

MRC . MAV

Your name SOLUTIONS

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Important note to candidates

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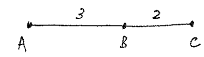
1. [5 marks]

Points **A** and **B** have position vectors $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 15 \\ 5 \end{pmatrix}$ respectively.

Find the point **C** such that **AB** : **AC** = 3 : 5.

$\vec{OC} = \vec{OA} + \frac{5}{3}\vec{AB}$ ✓



$= \underset{\sim}{a} + \frac{5}{3}(\underset{\sim}{b} - \underset{\sim}{a})$ ✓

$= -\frac{2}{3}\underset{\sim}{a} + \frac{5}{3}\underset{\sim}{b}$ ✓

$= -\frac{2}{3}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + \frac{5}{3}\begin{pmatrix} -1 \\ 15 \\ 5 \end{pmatrix}$ ✓

$= \begin{pmatrix} -3 \\ 23 \\ 9 \end{pmatrix}$ ✓

C is a point with position vector $-3\underset{\sim}{i} + 23\underset{\sim}{j} + 9\underset{\sim}{k}$

✓ states correct vector sum for $\vec{OC}$.
✓ determines $\vec{AB}$. ✓ correctly simplifies vector sum for $\vec{OC}$
✓ correctly uses vectors $\vec{OA}$ and $\vec{OB}$ in calculation.
✓ correctly calculates final answer.

5

2. [12 marks]

For each of the following functions, find $\frac{dy}{dx}$, expressing your answers in terms of x.

(a) $y = 2^{x^2} \cdot e^{\cos x}$

$\ln y = x^2 \cdot \ln 2 + \cos x \cdot \ln e$ ✓

$\frac{1}{y} \cdot \frac{dy}{dx} = 2x(\ln 2) + (-\sin x)$ ✓ ✓

$\frac{dy}{dx} = [x \cdot \ln 4 - \sin x] \cdot (2^{x^2} \cdot e^{\cos x})$ ✓

✓ takes ln of both sides.
✓✓ implicitly differentiates.
✓ multiplies both sides by y to give final answer.

[4]

(b) $y = \sqrt{\cos(\sin^2 x)}$

$y = [\cos(\sin^2 x)]^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}[\cos(\sin^2 x)]^{-1/2} \cdot [-\sin(\sin^2 x)] \cdot 2\sin x \cos x$ ✓ ✓ ✓

$= \dfrac{-\sin(\sin^2 x) \cdot \sin x \cos x}{\sqrt{\cos(\sin^2 x)}}$ ✓

[4]

8

(c) $\ln y = \dfrac{x}{x^2+1}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \dfrac{(x^2+1) - 2x^2}{(x^2+1)^2}$ ✓

$= \dfrac{1-x^2}{(x^2+1)^2}$ ✓

$\therefore \frac{dy}{dx} = \dfrac{1-x^2}{(x^2+1)^2} \cdot y$

$= \dfrac{1-x^2}{(x^2+1)^2} \cdot e^{\left(\frac{x}{x^2+1}\right)}$ ✓ ✓

[4]

4

3. [5 marks]

Use de Moivre's rule to determine the exact value of $(1+i)^5 - (1-i)^5$.

$1+i = \sqrt{2}\,\text{cis}\,\frac{\pi}{4}$, $1-i = \sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)$ ✓

$\therefore (1+i)^5 - (1-i)^5$

$= (\sqrt{2})^5\,\text{cis}\,\frac{5\pi}{4} - (\sqrt{2})^5\,\text{cis}\left(-\frac{5\pi}{4}\right)$ ✓

$= 4\sqrt{2}\left[\text{cis}\left(-\frac{3\pi}{4}\right) - \text{cis}\left(\frac{3\pi}{4}\right)\right]$ ✓

$= 4\sqrt{2}\left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right]$ ✓

$= 4\sqrt{2}\left[-\frac{2}{\sqrt{2}}i\right]$

$= \underline{\underline{-8i}}$ ✓

5

4. [4 marks]

For each of the following statements, circle either True or False.
If a statement is true, prove that it is true, showing your reasoning clearly.
If a statement is false, either explain why it is false, or provide an example to show the statement is false.

(a) The vector $c = -2i - 3j + k$ is perpendicular to both the vectors $a = -2i + 2j + 2k$ and $b = i - 3j - 7k$.

TRUE / FALSE

$\underset{\sim}{a} \cdot \underset{\sim}{c} = 4 - 6 + 2$
$= 0$ ✓

$\underset{\sim}{c} \cdot \underset{\sim}{b} = -2 + 9 - 7$
$= 0$ ✓

since $\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$ and $\underset{\sim}{b} \cdot \underset{\sim}{c} = 0$,
$\underset{\sim}{c}$ is $\perp$ to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

[2]

(b) If the matrices MN and NM are both defined, then the dimension of the matrices MN and NM is the same as the dimension of matrix M or matrix N.

TRUE / FALSE

For MN to be defined: dim(M) is $a \times b$
dim(N) is $b \times c$
$\therefore$ dim(MN) $= a \times c$

For NM to be defined: dim(NM) $= b \times b$, and $a = c$ ✓

$\Rightarrow$ dim(M) must be $c \times b$ and dim(N) is $b \times c$ ✓
and dim(MN) must be $c \times c$ and dim(NM) must be $b \times b$

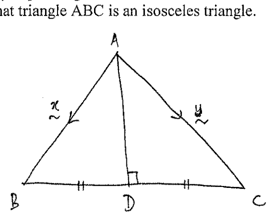
$\therefore$ Statement is false

[2] 4

6. [5 marks]

In triangle ABC, point D is on BC such that AD is perpendicular to BC and |DB| = |DC|. Let $\overrightarrow{AB} = x$ and $\overrightarrow{AC} = y$.

By expressing $\overrightarrow{AD}$ and $\overrightarrow{BC}$ as vectors in terms of x and y, use a vector method to prove that triangle ABC is an isosceles triangle.



✓

$\overrightarrow{BC} = \underset{\sim}{y} - \underset{\sim}{x}$

$\overrightarrow{AD} = \underset{\sim}{x} + \frac{1}{2}(\underset{\sim}{y} - \underset{\sim}{x})$

$= \frac{1}{2}\underset{\sim}{y} + \frac{1}{2}\underset{\sim}{x}$ ✓

$\overrightarrow{BC} \cdot \overrightarrow{AD} = (\underset{\sim}{y} - \underset{\sim}{x}) \cdot (\frac{1}{2}\underset{\sim}{y} + \frac{1}{2}\underset{\sim}{x}) = 0$ ✓

$\frac{1}{2}|\underset{\sim}{y}|^2 - \frac{1}{2}|\underset{\sim}{x}|^2 = 0$ ✓

$\Rightarrow |\underset{\sim}{y}| = |\underset{\sim}{x}|$

$\therefore |\overrightarrow{AB}| = |\overrightarrow{AC}|$ ✓

$\Rightarrow \Delta$ ABC is isosceles.

5

5. [6 marks]

Evaluate the following limits, showing full reasoning.

(a) $\lim_{x \to 0}\left(\dfrac{\sin\left(\frac{\pi}{2} + x\right) - 1}{x}\right)$

$= \frac{d}{dx}\left[\sin x\right]\Big|_{x=\pi/2}$ ✓

$= \left[\cos x\right]\Big|_{x=\pi/2}$ ✓

$= \cos\left(\frac{\pi}{2}\right)$

$= \underline{\underline{0}}$ ✓

[3]

(b) $\lim_{x \to 0}\dfrac{x\sqrt{12}}{\sin 3x}$

$= \lim_{x \to 0} \dfrac{3x}{\sin 3x} \cdot \dfrac{\sqrt{12}}{3}$ ✓ ✓

$= 1 \cdot \dfrac{\sqrt{12}}{3}$

$= \underline{\underline{\dfrac{2\sqrt{3}}{3}}}$ ✓

6 [3]

7. **[3 marks]**

There are four types of predators on the island that take chicks from the nest; cats, rats, lizards and gulls. The matrix P shows the proportion of chicks lost each day to each type of predator at each site.

$$P = \begin{array}{cccc} [0.015 & 0.01 & 0.005 & 0.018] \\ \text{cats} & \text{rats} & \text{lizards} & \text{gulls} \end{array}$$

The number of chicks at each nesting sites A, B and C in 2006 is given by the matrix

$$C = \begin{bmatrix} 10000 \\ 6500 \\ 9750 \end{bmatrix}$$

(a) Which of the matrix products PC or CP is defined? Explain why.

CP, since number of columns in C = number of rows in P ✓

[1]

(b) (i) Form the matrix product that is defined and call it R.

$$R = \begin{bmatrix} 150 & 100 & 50 & 180 \\ 97.5 & 65 & 32.5 & 117 \\ 146.25 & 97.5 & 48.75 & 175.5 \end{bmatrix}$$ ✓

(ii) Explain the meaning of the information that matrix R contains.

The number of chicks lost, in 2006, by each type of predator at each site. ✓

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[2]

END OF SECTION ONE

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MATHEMATICS SPECIALIST 3CD
Section Two
(Calculator Assumed)

Circle your teacher's initials

MRC MAV

Your name SOLUTIONS

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighters, eraser, ruler.

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

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1. **[6 marks]**

Determine the vector equation of the plane which passes through **A** <3, 2, 6>, **B** <1, –3, 10> and **C** <10, 0, 5>.

$\vec{AB} = \begin{pmatrix} -2 \\ -5 \\ 4 \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} 9 \\ 3 \\ -5 \end{pmatrix}$

let $\underset{\sim}{n}$ be vector perpendicular to the plane, and $\underset{\sim}{n} = \begin{pmatrix} a \\ b \\ 1 \end{pmatrix}$ ✓

$\underset{\sim}{n} \cdot \vec{AB} = 0$

$\Rightarrow -2a - 5b + 4 = 0$ —— ① ✓

and

$\underset{\sim}{n} \cdot \vec{BC} = 0$

$\Rightarrow 9a + 3b - 5 = 0$ —— ② ✓

Solving ① and ② simultaneously: $a = \frac{1}{3}$, $b = \frac{2}{3}$ ✓

$\therefore \underset{\sim}{n} = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$ or $\underset{\sim}{n} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ✓

So: $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 25$ ✓

6

2. **[6 marks]**

A plane passes through the point A (2, –3, 4) and has a normal vector of $\begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix}$.

A line passes through **B** (16, –17, –8) and is parallel to the vector $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

Determine where the line and plane intersect.

$\underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix}$

Eqn of plane: $\underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} = -5$ ✓

$\underset{\sim}{r} = \begin{pmatrix} 16 \\ -17 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

Eqn of line: $\underset{\sim}{r}_2 = \begin{pmatrix} 16 - 2\lambda \\ 3\lambda - 17 \\ \lambda - 8 \end{pmatrix}$ ✓

At intersection point, $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$ ✓

$\therefore \begin{pmatrix} 16 - 2\lambda \\ 3\lambda - 17 \\ \lambda - 8 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix} = -5$ ✓

$2\lambda - 16 + 15\lambda - 85 + 3\lambda - 24 = -5$

$\lambda = 6$ ✓

$\therefore$ point of intersection is $(4, 1, -2)$ ✓

6

3. [4 marks]

The system of equations

$2x + y + 7z = 9556$
$3x + y + 4z = 5899$
$5x + 2y + z = 3155$

can be used to estimate the number of cats (x), rats (y), and lizards (z), on an island used as a nature reserve.

(a) Write this system of simultaneous linear equations in matrix form.

$$\begin{pmatrix} 2 & 1 & 7 \\ 3 & 1 & 4 \\ 5 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9556 \\ 5899 \\ 3155 \end{pmatrix} \checkmark$$

[1]

(b) Write down the inverse matrix that can be used to solve this system of simultaneous linear equations.

$$\begin{pmatrix} 2 & 1 & 7 \\ 3 & 1 & 4 \\ 5 & 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -0.7 & 1.3 & -0.3 \\ 1.7 & -3.3 & 1.3 \\ 0.1 & 0.1 & -0.1 \end{pmatrix} \checkmark$$

[1]

(c) Solve the system of simultaneous linear equations and hence estimate the number of cats, rats and lizards on the island.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -0.7 & 1.3 & -0.3 \\ 1.7 & -3.3 & 1.3 \\ 0.1 & 0.1 & -0.1 \end{pmatrix}\begin{pmatrix} 9556 \\ 5899 \\ 3155 \end{pmatrix}$$

$$= \begin{pmatrix} 33 \\ 880 \\ 1230 \end{pmatrix} \checkmark$$

[2]

$\therefore$ 33 cats, 880 rats, 1230 lizards ✓

4

(c) Determine the matrix M which satisfies the equation:

$$MZ - \frac{1}{4}M = 2I$$

where I is the 2×2 identity matrix.

$$M\left(Z - \tfrac{1}{4}I\right) = 2I \checkmark$$

$$M = 2I\left(Z - \tfrac{1}{4}I\right)^{-1} \checkmark$$

$$= 2\left[\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}\right]^{-1} \checkmark$$

$$= 2\begin{bmatrix} 3/4 & 2 \\ 0 & 3/4 \end{bmatrix}^{-1}$$

$$= 2\begin{bmatrix} 4/3 & -32/9 \\ 0 & 4/3 \end{bmatrix}$$

$$\therefore \quad M = \begin{bmatrix} \frac{8}{3} & -\frac{64}{9} \\ 0 & \frac{8}{3} \end{bmatrix} \checkmark$$

4

[4]

4. [12 marks]

Let $W = \begin{bmatrix} d-2 & -3 \\ -1 & d \end{bmatrix}$, $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $Y = \begin{bmatrix} -1 & -4 \end{bmatrix}$ and $Z = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

(a) Evaluate each of the following where possible. If not possible, state this clearly and indicate the reason for your decision.

(i) WX

$$= \begin{bmatrix} d-2 & -3 \\ -1 & d \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2d-4-9 \\ -2+3d \end{bmatrix} \checkmark$$

$$= \begin{bmatrix} 2d-13 \\ 3d-2 \end{bmatrix} \checkmark$$

[2]

(ii) 2Y + Z

Not possible, since the dimensions of Y and Z are unequal. ✓ ✓

[2]

(b) Determine W^{-1}, stating all the necessary restrictions on d so that W^{-1} exists.

$$W^{-1} = \frac{1}{d(d-2)-3}\begin{bmatrix} d & 3 \\ 1 & d-2 \end{bmatrix} = \frac{1}{(d^2-2d-3)}\begin{bmatrix} d & 3 \\ 1 & d-2 \end{bmatrix} \checkmark \checkmark$$

For W^{-1} to exist,

$$d^2 - 2d - 3 \neq 0 \checkmark$$

$$(d+1)(d-3) \neq 0$$

$$d \neq -1 \text{ and } d \neq 3 \checkmark$$

[4]

8

5. [6 marks]

Use the method of proof by exhaustion to prove that every integer which is a perfect cube is either a multiple of 9, or 1 more, or 1 less than a multiple of 9.

Let integer n be represented by $3k$, $3k-1$ and $3k+1$ ✓

If $n = 3k$:
$$n^3 = (3k)^3 = 27k^3 = 9(3k^3) \Rightarrow \text{a multiple of } 9. \checkmark$$

If $n = 3k-1$:
$$n^3 = (3k-1)^3$$
$$= 27k^3 - 27k^2 + 9k - 1 \checkmark$$
$$= 9(3k^3 - 3k^2 + k) - 1 \Rightarrow 1 \text{ less than a multiple of } 9. \checkmark$$

If $n = 3k+1$:
$$n^3 = (3k+1)^3$$
$$= 27k^3 + 27k^2 + 9k + 1 \checkmark$$
$$= 9(3k^3 + 3k^2 + 3k) + 1 \Rightarrow 1 \text{ more than a multiple of } 9. \checkmark$$

6

6. **[7 marks]**

(a) Solve the inequality $|x + 2| < |3 - x|$.

ON Calculator:

$x < \frac{1}{2}$

ALGEBRA:

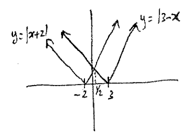
$(x+2)^2 < (3-x)^2$

$x^2 + 4x + 4 < 9 - 6x + x^2$

$10x < 5$

$x < \frac{1}{2}$

Graph:

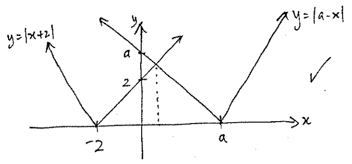


$x < \frac{1}{2}$

✓✓

[2]

(b) Hence, or otherwise, solve the inequality $|x + 2| < |a - x|$ for all real values of a.



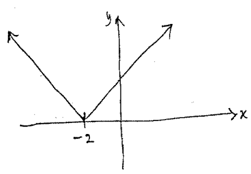
✓

Intersection point: $x + 2 = a - x$

$x = \frac{a}{2} - 1$ ✓ for $a \neq -2$

$\therefore$ $x < \frac{a}{2} - 1$ (for $a \neq -2$) ✓

For $a = -2$: ✓ The graphs of $y = |x+2|$ and $y = |-2-x|$ coincide



$\therefore$ NO solution for $a = -2$ ✓

[5]

7

7. **[5 marks]**

Determine the equation of the tangent to the curve defined by $y^3 + 2xy = 9$ at the point P whose coordinates are (4, 1).

$y^3 + 2xy = 9$

$3y^2\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) + 2y = 0$ ✓

$\frac{dy}{dx}\left(3y^2 + 2x\right) = -2y$

$\therefore \frac{dy}{dx} = \frac{-2y}{3y^2 + 2x}$ ✓

At $(4,1)$, $\frac{dy}{dx} = \frac{-2}{11}$ ✓

So, $y - 1 = -\frac{2}{11}(x - 4)$ ✓

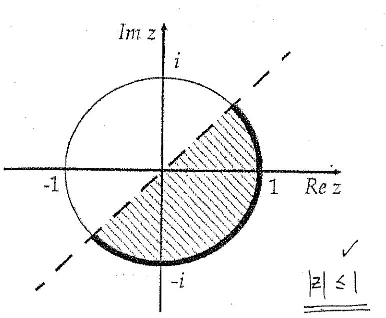
$2x + 11y = 19$ ✓

OR $y = -\frac{2x}{11} + \frac{19}{11}$

5

8. **[7 marks]**

(a) Write down the inequalities of the complex number z such that together they describe the set of points shaded below.



✓

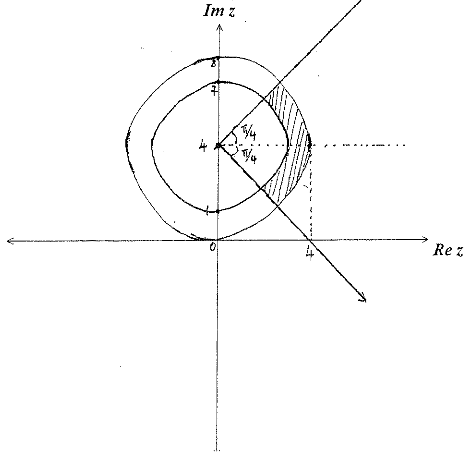
$|z| \leq 1$ and $-\frac{3\pi}{4} < Arg(z) < \frac{\pi}{4}$

limits ✓
correct signs ✓

[3]

(b) On the Argand plane provided, indicate the region defined by:

$\{z : 3 \leq |z - 4i| \leq 4\} \cap \{z : -\frac{\pi}{4} \leq Arg(z - 4i) \leq \frac{\pi}{4}\}$



centre at $4i$ ✓
two rays ✓
two circles ✓
correct area shaded ✓

[4] 7

9. **[8 marks]**

Two fighter jets are on a practice flight. At time t = 0 seconds, Jet A is at position $\begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$ km and jet B is at position $\begin{pmatrix} 3 \\ 7 \\ -2 \end{pmatrix}$ km. Jet A is flying at a velocity of $\begin{pmatrix} 200 \\ 350 \\ 450 \end{pmatrix}$ m/s while jet B has a velocity of $\begin{pmatrix} -300 \\ -450 \\ 250 \end{pmatrix}$ m/s.

(a) At what time are the two jets closest to each other, to the nearest 0.01 second?

$\underset{\sim}{r}_A(t) = \begin{pmatrix} 4 + 0.2t \\ -5 + 0.35t \\ 6 + 0.45t \end{pmatrix}$ ✓ , $\underset{\sim}{r}_B(t) = \begin{pmatrix} 3 - 0.3t \\ 7 - 0.45t \\ -2 + 0.25t \end{pmatrix}$ ✓

$\therefore {}_A\underset{\sim}{r}_B(t) = \begin{pmatrix} 1 + 0.5t \\ -12 + 0.8t \\ 8 + 0.2t \end{pmatrix}$ ✓

${}_A\underset{\sim}{v}_B = \begin{pmatrix} 0.5 \\ 0.8 \\ 0.2 \end{pmatrix}$ ✓

For closest approach: ${}_A\underset{\sim}{r}_B(t) \cdot {}_A\underset{\sim}{v}_B = 0$ ✓

$\begin{pmatrix} 1 + 0.5t \\ -12 + 0.8t \\ 8 + 0.2t \end{pmatrix} \cdot \begin{pmatrix} 0.5 \\ 0.8 \\ 0.2 \end{pmatrix} = 0$

$t = 8.06$ sec (2 dp) ✓

[6]

6

(b) Calculate the shortest distance between the two jets, correct to the nearest metre.

$$\left| {}_{A}\underset{\sim}{r}_{B}(8.06) \right| = 12.1867 \text{ km} \checkmark$$

$$= \underline{\underline{12\,187}} \text{ (nearest metre)} \checkmark$$

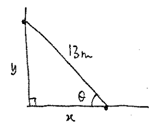
[2]

2

10. [9 marks]

A ladder 13 metres long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate of 0.5 m/sec, and the bottom of the ladder is 5 metres from the wall,

(a) how fast is the top of the ladder sliding down the wall, correct to 2 decimal places?



✓

For 13m ladder, $x=5$, $y=12$ ✓

$$x^2 + y^2 = 13 \checkmark$$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0 \checkmark$$

$$2(5)(0.5) + 2(12)\left(\frac{dy}{dt}\right) = 0$$

$$\therefore \frac{dy}{dt} = -0.208\dot{3} \text{ m/s}$$

Ans: sliding down at $\underline{\underline{0.21 \text{ m/sec}}}$ (2 dp) ✓

[5]

(b) at what rate, in degrees per second, is the angle between the ladder and the ground changing?

$$\cos\theta = \frac{x}{13}$$

$$\therefore x = 13\cos\theta \checkmark$$

$$\frac{dx}{dt} = -13\sin\theta\left(\frac{d\theta}{dt}\right) \checkmark$$

$$0.5 = -13\left(\frac{12}{13}\right)\left(\frac{d\theta}{dt}\right)$$

$$\therefore \frac{d\theta}{dt} = -\frac{1}{24} \text{ rad/sec} \checkmark$$

$$= \underline{\underline{2.4^\circ/\text{sec}}} \checkmark$$

[4]

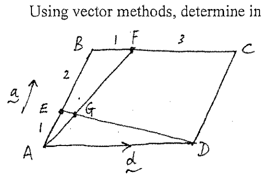
9

11. [10 marks]

ABCD is a parallelogram with points E and F such that AE : EB = 1 : 2 and BF : FC = 1 : 3. G is the point where AF and ED intersect.

Let Let $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{AD} = \boldsymbol{d}$.

Using vector methods, determine in what ratios AF and ED intersect each other.



✓

let $\overrightarrow{AG} = k\overrightarrow{AF}$

and $\overrightarrow{EG} = h\overrightarrow{ED}$ ✓

$$\overrightarrow{AF} = \underset{\sim}{a} + \tfrac{1}{4}\underset{\sim}{d} \qquad \overrightarrow{ED} = \underset{\sim}{d} - \tfrac{1}{3}\underset{\sim}{a}$$

$$\therefore \overrightarrow{AG} = k\underset{\sim}{a} + \tfrac{k}{4}\underset{\sim}{d} \checkmark \qquad \therefore \overrightarrow{EG} = h\underset{\sim}{d} - \tfrac{h}{3}\underset{\sim}{a} \checkmark$$

$$\overrightarrow{AG} = \overrightarrow{AE} + \overrightarrow{EG} \checkmark$$

$$k\underset{\sim}{a} + \tfrac{k}{4}\underset{\sim}{d} = \tfrac{1}{3}\underset{\sim}{a} + h\underset{\sim}{d} - \tfrac{h}{3}\underset{\sim}{a}$$

$$k\underset{\sim}{a} + \tfrac{k}{4}\underset{\sim}{d} = \left(\tfrac{1}{3} - \tfrac{h}{3}\right)\underset{\sim}{a} + h\underset{\sim}{d} \checkmark$$

$$\therefore k = \tfrac{1}{3} - \tfrac{h}{3} \quad \text{and} \quad \tfrac{k}{4} = h$$

$$k = 4h$$

$$\Rightarrow 12h = 1 - h$$

$$h = \tfrac{1}{13} \checkmark \text{ and } k = \tfrac{4}{13} \checkmark$$

∴ G divides $\underline{\text{ED in the ratio } 1:12}$ ✓

and divides $\underline{\underline{\text{AF in the ratio } 4:9}}$ ✓

END OF SECTION TWO

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